Calculation of impinging-jet heat transfer with the low-Reynolds-number q–ζ turbulence model

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A two-equation model of turbulent heat transfer has been devised along the lines of the successful q–ζ model of the turbulent velocity field. The new variables are the square root of the temperature variance and its dissipation rate. These variables are attractive for eddy-diffusivity calculations, because their variation with distance y from the wall is linear in y when $y \to 0$. This feature in the q–ζ model for the eddy-viscosity leads to economical computations of good accuracy. The new heat transfer model has been calibrated with data from channel and boundary-layer flows and is applied here to the computation of heat transfer in the stagnation region of an impinging jet. The results compare very favourably with those based upon the assumption of a constant turbulent Prandtl number. This quantity is obtained as an output from the calculations instead of being an input to them.

Keywords: turbulence; modelling; heat transfer; CFD

Introduction

The k–ε turbulence model owes its pre-eminent position in computational fluid dynamics to the discovery by Spalding (1969) and his contemporaries that there was no need to modify the length-scale equation (in this case the ε-equation) in order to handle wall flows, provided always that the problem of resolving the low Reynolds number turbulence close to the wall was avoided by the use of "wall functions." For this reason, the promising k–W and k–kL models of Gibson and Spalding (1972) and Ng and Spalding (1972) were discarded in favour of a related form which appeared at the time to be computationally more convenient. Although the high-Reynolds-number k–ε model has given good service over the years, it is now difficult to justify the continued use of wall functions, particularly in applications where the existence of universal wall laws is doubtful and as arguments for computational economy become weaker. Wall functions endure mainly because the problem of integrating the turbulence equations to the wall has yet to be satisfactorily solved. The choice of ε as a variable adds to the difficulties: it has no natural boundary condition at the wall, so that one must be derived from assumed limiting behaviour of the turbulence fluctuations as $y \to 0$. Moreover, if the assumed wall value ε_w is subtracted to form a new variable, $\tilde{\varepsilon} = \varepsilon - \varepsilon_w$, which is zero at the wall, this quantity is found to vary as y^2 as $y \to 0$, thus necessitating the use of an excessively fine grid for computations through the sublayers, as well as the need to calculate spatial derivatives of the turbulence energy k. Other "wall" terms are also needed, whose exact form is uncertain, and which destroy the attractive simplicity of the high-Reynolds-number model, which is one of its chief merits. There is, in short, no advantage in persisting with the ε-equation except the many years of experience built into it, much of which may still be employed in the formulation of more useful equations.

Alternatives include equations for the turbulent vorticity $\omega \equiv \varepsilon/k$ and its reciprocal, the time-scale $\tau \equiv k/\varepsilon$. And there are other variables fitting the general form of $k^m\varepsilon^n$. Spalding (1991) has pointed out that the question of which of these is best has never been seriously investigated. We have found that there is much to be said for the use of new variable $\zeta \equiv \tilde{\varepsilon}/2\sqrt{k}$ especially when the k-equation is replaced by one for $q \equiv \sqrt{k}$. ζ is then identified as the dissipation rate of q. Both variables are zero at the wall and are well behaved in that they vary linearly with distance from it for small y. ζ lies between ω and ε in the progression $\omega \equiv \tilde{\varepsilon}/k$, $\zeta \equiv \tilde{\varepsilon}/2\sqrt{k}$ and $\tilde{\varepsilon}$, meeting the empirical condition that for simplicity in the high-Reynolds number form the exponent of $\tilde{\varepsilon}$ in the general variable above should be unity. $\zeta \propto \sqrt{(\omega\varepsilon)}$ apparently acquires the merits of the earlier models without, it seems, many of the defects. The motivation for a new model was computational efficiency. The improved accuracy shown in the attached and separated flow calculations by Gibson and Dafa'Alla (1995) and Gibson and Harper (1995) was an unexpected bonus.

In heat transfer calculations with two-equation models the eddy-diffusivity is usually related to the eddy-viscosity via a turbulent Prandtl number $\mathrm{Pr}_t \equiv \nu_t/\alpha_t$, taken to be a constant ≈ 0.9 for wall flows. There is then no need to define the eddy-diffusivity in terms of the variables of the scalar field: by analogy with k and ε, the (half) variance $k_\vartheta \equiv \overline{\vartheta^2}/2$ and its dissipation rate ε_ϑ. Even when the equations are integrated to the wall the constant Pr_t assumption may still be useful in some cases, but it cannot be relied upon generally. In particular it

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breaks down for the computation of heat transfer at a stagnation point, as in the reattaching flow downstream of a sudden pipe expansion, or at the impingement point of a jet on a plate. In these regions, the calculated heat transfer rates may exceed the actual values by a large margin unless they are reduced by correction factors like that of Yap (Craft et al. 1993). The alternative is to detach the eddy-diffusivity α_t from its fixed ratio to the eddy-viscosity, and model it separately with the aid of the scalar-field equations.

In integrating the equations for k_ϑ and ε_ϑ to the wall, the difficulty arises, as before, in the specification of the correct boundary conditions for these variables. Examination of the limiting behaviour at the wall shows that the possibility of a non-zero wall value of k_ϑ cannot be excluded. Sommer et al. (1993) provide an interesting discussion of this point. If, however, k_ϑ is zero at the wall, its variation with y^2, like that of k, will still be inconvenient. It is proposed, therefore, to replace the equation for k_ϑ by one for $q_\vartheta \equiv \sqrt{k_\vartheta}$, the counterpart of q, and the equation for ε_ϑ by one for ζ_ϑ, the dissipation rate of q_ϑ. These variables, like q and ζ, possess the useful property of varying linearly at the wall (as long as the wall temperature fluctuations can be neglected) and thereby further facilitate economical computations. The ζ_ϑ equation is obtained by transforming a suitable modelled ε_ϑ equation in the same way as Gibson and Dafa'Alla (1995) derived in ζ-equation from the ε-equation. The presence of both velocity and temperature time scales, k/ε and $k_\vartheta/\varepsilon_\vartheta$, in the equations is a complicating factor. A variety of composite scales have been suggested by different authors. In this, as in the use of damping functions for integration to the wall, we have drawn upon the experience of others in k_ϑ–ε_ϑ modelling, adapted as required to the q_ϑ–ζ_ϑ format.

The impinging axisymmetric jet is an interesting flow in practice as well as providing a demanding test case for turbulence modelling. Craft et al. (1993) summarise the main flow characteristics as: nearly irrotational normal straining in the vicinity of the stagnation point, strong streamline curvature nearer the free edge and, in the flow parallel to the plate, wall-jet behaviour in which the zero of the shear stress does not coincide with the zero of the velocity gradient. Because of this complexity, the more recent attempts at flow prediction have chiefly used second-moment closures. No doubt the effects of strong streamline curvature are best handled at a superior level of closure, as is the development of the wall jet, but we are concerned here primarily with the prediction of heat transfer in the vicinity of the stagnation point where, according to Craft et al., "the most important factor in determining the local Nusselt number is the distribution of turbulent flux across the sublayer or, when an eddy-viscosity model is used in this region, the distribution of turbulent thermal diffusivity." One of the four models assessed by Craft et al. was the Launder and Sharma (1974) low-Reynolds number k–ε model with uniform turbulent Prandtl number and incorporating the Yap correction in order to prevent high predicted turbulence levels causing excessive heat flux in the stagnation region. Dianat et al. (1995) find that for this flow a second-moment closure is generally superior to the "standard" k–ε model with wall functions.

Turbulence Modelling

The q–ζ eddy-viscosity model

The variables in the q–ζ model are defined as $q \equiv \sqrt{k}$ and its rate of destruction $\zeta \equiv \tilde{\varepsilon}/2q$, where $\tilde{\varepsilon}$ is the so-called "isotropic dissipation" introduced by Jones and Launder (1972) in recognition of the "decisive advantages" of a dependent variable which is zero at the wall. The accompanying drawback is the need to calculate $(\partial q/\partial x_j)^2$ in order to recover the true dissipation rate from the definition:

$$\varepsilon = \tilde{\varepsilon} + 2\nu\left(\frac{\partial q}{\partial x_j}\right)\left(\frac{\partial q}{\partial x_j}\right) \tag{1}$$

This calculation is unnecessary in the q–ζ model.

The first steps is to derive the q-equation from the modelled k-equation as:

$$U_i\frac{\partial q}{\partial x_i} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_q}\right)\frac{\partial q}{\partial x_j}\right] + Q - \zeta \tag{2}$$

where $Q \equiv P/2q$ and P is the production rate of k. Note, however, that the equations are not exact transforms of each other, because the turbulent transport terms do not exactly correspond. The assumption of simple gradient diffusion in each case necessarily involves the neglect of second-order terms in the transformation and thus introduces slightly different physics. It is argued that the gradient-diffusion assumption for q is no less reasonable than it is for k. Wilcox (1993) discusses this point in connexion with differences between the k–ε and k–ω models.

The ζ equation is obtained by transforming the k- and ε-equations as follows:

$$U_i\frac{\partial \zeta}{\partial x_i} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\zeta}\right)\frac{\partial \zeta}{\partial x_j}\right] + \frac{\zeta}{q}(C_{\zeta 1}f_{\zeta 1}Q - C_{\zeta 2}f_{\zeta 2}\zeta) + \psi' \tag{3}$$

in which $C_{\zeta 1}$, $C_{\zeta 2}$, $f_{\zeta 1}$, and $f_{\zeta 2}$ are constants and damping functions which are related to terms in the ε-equation by $C_\zeta f_\zeta = (2C_\varepsilon f_\varepsilon - 1)$. The assumption of gradient diffusion for the turbulent transport of ζ again involves the justifiable neglect of second-order terms in the transformation with consequence small changes in the physics. In Equation 3 ψ' accounts for secondary mean-flow production of ζ which, for the time being, is modelled as in Launder and Sharma (1974):

$$\psi' = \frac{\nu\nu_t}{q}\left(\frac{\partial^2 U_i}{\partial x_j \partial x_k}\right)\left(\frac{\partial^2 U_i}{\partial x_j \partial x_k}\right) \tag{4}$$

The eddy-viscosity is:

$$\nu_t \equiv C_\mu f_\mu \frac{q^3}{2\zeta} \tag{5}$$

where f_μ is the viscous-layer damping function for which the established Launder and Sharma formula is preferred to alternative expressions, because it depends only on the turbulence Reynolds number $R_t \equiv k^2/\nu\varepsilon$, and its use avoids the problems of specifying wall distance in complex flows.

$$f_\mu = \exp\left[\frac{-A_\mu}{(1 + R_t/50)^2}\right] \tag{6}$$

with $A_\mu = 6.0$ substituted for the Launder–Sharma value of 3.4 so as to give the best results for channel and boundary-layer flows. For the time being the standard k–ε model constants $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$ are used to give $C_{\zeta 1} = 1.88$, $C_{\zeta 2} = 2.84$, $\sigma_q = 1.0$, and $\sigma_\zeta = \sigma_\varepsilon = 1.3$.

The q_ζ–ζ_ϑ model

When the temperature fluctuations are zero at a wall, the limiting variation of the variance $\overline{\vartheta^2}$ is as y^2. A linear variation is clearly preferable on computational grounds, and for that reason, we define a new variable $q_\vartheta \equiv \sqrt{k_\vartheta}$ whose equation is readily derived from the $\overline{\vartheta^2}$-equation as:

$$U_i \frac{\partial q_\vartheta}{\partial x_i} = \frac{\partial}{\partial x_j}\left(\alpha \frac{\partial q_\vartheta}{\partial x_j} - \overline{u_j q'_\vartheta}\right) + \frac{P_\vartheta}{2q_\vartheta} - \frac{\varepsilon_\vartheta}{2q_\vartheta} + \frac{\alpha}{q_\vartheta}\frac{\partial q_\vartheta}{\partial x_j}\frac{\partial q_\vartheta}{\partial x_j} \tag{7}$$

where:

$$P_\vartheta \equiv -\overline{u_j \vartheta}\frac{\partial T}{\partial x_j} \quad \text{and} \quad \varepsilon_\vartheta \equiv \alpha \overline{\frac{\partial \vartheta}{\partial x_j}\frac{\partial \vartheta}{\partial x_j}} \tag{8}$$

If the variation near the wall of the fluctuating part of the temperature is:

$$\vartheta = \vartheta_w + ay + by^2 + cy^3 + \cdots \tag{9}$$

where the wall value ϑ_w and the coefficients $a, b, c, \ldots$ are random functions of time, the dissipation rate at the wall is obtained as:

$$\frac{\varepsilon_\vartheta}{\alpha} \to \overline{\frac{\partial \vartheta}{\partial y}\frac{\partial \vartheta}{\partial y}} = \overline{a^2} + 4\overline{ab}y + (4\overline{b^2} + 6\overline{ac})y^2 + \ldots \tag{10}$$

For the case $\vartheta = \vartheta_w = 0$ and $y = 0$, it is then easy to show that:

$$\frac{\partial q_\vartheta}{\partial x_j}\frac{\partial q_\vartheta}{\partial x_j} = \frac{\overline{a^2}}{2} + 2\overline{ab}y + \ldots \tag{11}$$

and so, at any rate to first order, the total dissipation-rate term in the q_ϑ equation:

$$\frac{1}{2q_\vartheta}\left(\varepsilon_\vartheta - 2\alpha \frac{\partial q_\vartheta}{\partial x_j}\frac{\partial q_\vartheta}{\partial x_j}\right)$$

is zero and $O(y)$ at the wall. We call this quantity ζ_ϑ and derive a transport equation for it by transforming an established modelled ε_ϑ-equation of the following general form containing thermal and mechanical time scales:

$$U_i \frac{\partial \varepsilon_\vartheta}{\partial x_i} = \frac{\partial}{\partial x_j}\left\{\left(\alpha + \frac{\alpha_t}{\sigma_{\varepsilon\vartheta}}\right)\frac{\partial \varepsilon_\vartheta}{\partial x_j}\right\} + C_{\vartheta 1} f_{\vartheta 1}\frac{\varepsilon_\vartheta}{k_\vartheta}P_\vartheta + C_{\vartheta 2} f_{\vartheta 2}\frac{\varepsilon}{k}P_\vartheta$$
$$+ C_{\vartheta 3} f_{\vartheta 3}\frac{\varepsilon_\vartheta}{k}P - C_{\vartheta 4} f_{\vartheta 4}\frac{\varepsilon_\vartheta}{k_\vartheta}\varepsilon_\vartheta - C_{\vartheta 5} f_{\vartheta 5}\frac{\varepsilon}{k}\varepsilon_\vartheta + \psi_\vartheta \tag{12}$$

The Cs and $\sigma_{\varepsilon\vartheta}$ are constants in the high-Reynolds-number form, and the fs are low-Reynolds-number damping functions. The ζ_ϑ-equation is obtained from:

$$\frac{D}{Dt}\left(\frac{\tilde{\varepsilon}_\vartheta}{2q_\vartheta}\right) = \frac{1}{2q_\vartheta}\frac{D\tilde{\varepsilon}_\vartheta}{Dt} - \frac{\tilde{\varepsilon}_\vartheta}{2q_\vartheta^2}\frac{Dq_\vartheta}{Dt} \tag{13}$$

as:

$$U_i \frac{\partial \zeta_\vartheta}{\partial x_i} = \frac{\partial}{\partial x_j}\left\{\left(\alpha + \frac{\alpha_t}{\sigma_{\zeta\vartheta}}\right)\frac{\partial \zeta_\vartheta}{\partial x_j}\right\} + (2C_{\vartheta 1} f_{\vartheta 1} - 1)\frac{\zeta_\vartheta}{q_\vartheta}Q_\vartheta$$
$$+ 2C_{\vartheta 2} f_{\vartheta 2}\frac{\zeta}{q}Q_\vartheta + 2C_{\vartheta 3} f_{\vartheta 3}\frac{\zeta_\vartheta}{q}Q - (2C_{\vartheta 4} f_{\vartheta 4} - 1)$$
$$\times \frac{\zeta_\vartheta}{q_\vartheta}\zeta_\vartheta - 2C_{\vartheta 5} f_{\vartheta 5}\frac{\zeta}{q}\zeta_\vartheta + \psi'_\vartheta \tag{14}$$

where $Q \equiv P/2q$ and $Q_\vartheta \equiv P_\vartheta/2q_\vartheta$. We have used the received values of the ε_ϑ-equation model constants as chosen by Nagano and Kim (1988) and Sommer et al. (1992): $C_{\vartheta 1} = 0.90$, $C_{\vartheta 2} = 0$, $C_{\vartheta 3} = 0.72$, $C_{\vartheta 4} = 1.10$, $C_{\vartheta 5} = 0.80$; $\sigma_{q\vartheta} = \sigma_{\zeta\vartheta} = 1.0$. Guided by the precedent of the ε or ζ equations, in which low-Reynolds-number damping is unnecessary in the production terms, we have followed the example of Sommer et al. (1992) in setting $f_{\vartheta 1}$, $f_{\vartheta 2}$, and $f_{\vartheta 3}$ all equal to unity. In the ε or ζ equations, the dissipation term damping function is chosen to reproduce the decay of grid turbulence in the final period. The decay of heated grid turbulence is a vexed subject; the observed decay of $\overline{\vartheta^2}$ cannot be fitted by simple relationships of the sort that suffice for k. And since the inclusion of a final period decay term has little effect on near-wall calculations of the velocity field anyway, we are content to omit them in the q_ϑ–ζ_ϑ model and put $f_{\vartheta 4} = f_{\vartheta 5} = 1.0$.

The low-Reynolds-number treatment is then confined to damping the eddy-diffusivity coefficient and specifying the term ψ'_ϑ in Equation 14. In a straightforward formulation of the type $\alpha_t \propto q^2 t^*$, the time scale t^* is conventionally formed from the mechanical and thermal scales, $t \equiv q/2\zeta$, $t_\vartheta \equiv q_\vartheta/2\zeta_\vartheta$. Thus, Elghobashi and Launder (1983), in their study of the development of a thermal mixing layer, used an ε_ϑ-equation involving the composite scale $t^* \equiv \sqrt{(t t_\vartheta)}$. Dakos and Gibson (1993) deduced $t^* = (1/t + 1/t_\vartheta)^{-1}$ from measurements in homogeneous turbulence with temperature gradient. Abe et al. (1995) present persuasive arguments for a hybrid scale on these lines:

$$\frac{1}{t^*} = \frac{1}{2}\left(\frac{1}{t} + \frac{C_m}{t_\vartheta}\right) = \frac{1}{t}\left(\frac{\mathrm{R} + C_m}{2\mathrm{R}}\right) \tag{15}$$

where $\mathrm{R} \equiv t_\vartheta/t$. Constant C_m is set equal to 0.5 so as to give $\mathrm{Pr}_t \approx 0.9$ in the logarithmic wall layer. The eddy-diffusivity is then expressed as:

$$\alpha_t = C_\lambda f_\lambda \frac{q^3}{2\zeta}\left(\frac{2\mathrm{R}}{C_m + \mathrm{R}}\right) \tag{16}$$

with $C_\lambda = 0.10$.

There are widely recognised advantages in avoiding the use of the distance from the wall in formulating the damping function. Thus the Launder and Sharma (1974) Reynolds number function retains its popularity despite its recognised failure to capture any pressure-reflection effect or to conform precisely to limiting conditions at the wall. It turns out, however, that this sort of function alone just does not work for the temperature field, and we have had to fall back on a y-dependent function for the time being. The form devised and tested by Abe et al. (1995):

$$f_\lambda = \left[1 - \exp\left(-\frac{y^*}{14}\right)\right]\left[1 - \exp\left(-\frac{\mathrm{Pr}^{1/2} y^*}{14}\right)\right] \tag{17}$$

where $y^* \equiv y/\eta$, and η is the Kolmogorov scale $(\nu^3/\varepsilon)^{1/4}$. For the remaining source term in Equation 14:

$$\psi'_\vartheta = \frac{\alpha\alpha_t(1 - f_\lambda)}{2q_\vartheta}\left(\frac{\partial^2 T}{\partial x_j \partial x_k}\right)\left(\frac{\partial^2 T}{\partial x_j \partial x_k}\right) \tag{18}$$

This expression, from Nagano et al. (1994), has half the strength of the equivalent term in the ζ-equation, and the inclusion of f_λ ensures that it falls quickly to negligible levels outside the viscous layers.

The model has been calibrated with the aid of direct simulations of channel flow, and it has been found to give generally good results for heat transfer through boundary layers in zero and positive pressure gradients. Figures 1–3 show predictions of the mean and rms temperature profiles, and that of the temperature flux $\overline{v\vartheta}$, which agree closely with the DNS results of Kasagi et al. (1992) for fully developed channel flow at low Reynolds number.

The impinging jet

The q–ζ and q_ϑ–ζ_ϑ models have been used to calculate the impinging jet studied experimentally by Cooper et al. (1993) and as a test case for turbulence models by Craft et al. (1993). Use has also been made of the heat transfer data acquired by Baughn and Shimizu (1989) and Baughn et al. (1992). The flow arrangement is shown in Figure 4. A turbulent air jet emerges with mean velocity U_B from a pipe of (internal) diameter D to impinge at right angles on a heated plane positioned a distance $H = 2D$ below. In the two experiments reported, of diameters of 26 and 101.5 mm were used. In each case, the flow in the jet exit plane was fully developed, and the Reynolds number $U_B D/\nu \approx 23{,}000$. Hot-wire measurements of mean flow and turbulence quantities extended to radial distances up to $9D$. Baughn and Shimizu and Baughn et al. obtained heat transfer data from similar experiments with a heated plate.

The computational domain shown in Figure 4 extended $10D$ in the radial r-direction and $2.5D$ in the y-direction normal to the plate. Fully developed pipe flow was assumed in the jet exit plane with conditions obtained from a separate calculation. On the entrainment boundaries in the r- and y-directions, the mean velocity normal to the boundary was calculated from the continuity condition on the assumption of constant static pressure. Turbulence quantities q and ζ were assigned zero values on these boundaries in regions of inward flow. In outflow regions, zero-gradient conditions were applied. Standard numerical methods were used with coordinate-based grids, staggered for velocity and pressure, hybrid differencing, and pressure correction using the SIMPLE algorithm. To ensure grid-independent results, calculations were performed on several grids consisting of up to 112 and 186 nodes in the y- and r-directions, respectively, with high nodal concentrations at the jet pipe wall in the exit plane, in the stagnation zone, and in the near-wall low-Reynolds-number region. The first near-wall nodes were located in the sublayer, typically at $y^+ \approx 1$.

Comparative calculations with the q–ζ and k–ε models were made on the same computational grid, starting from the same initial conditions. We were less concerned in these calculations to demonstrate the superior computational efficiency of the q–ζ model than to effect the comparison under the same conditions. It was convenient to use for the q–ζ calculations the grid refined to achieve grid-independent solutions with the k–ε model. Questions of grid optimisation and computational efficiency are dealt with by Harper (1995) in connexion with the flows over backward-facing steps. In those calculations, grid-independent solutions with the q–ζ model were obtained with roughly 30% fewer nodes than were needed for the k–ε model. Furthermore, q–ζ calculations tend always to be more stable and less sensitive to initial conditions. A relatively rapid convergence rate levels out so that for good accuracy, overall rates of convergence are approximately the same in each case. The gains in efficiency, though well worth having, are significantly less than the 50%

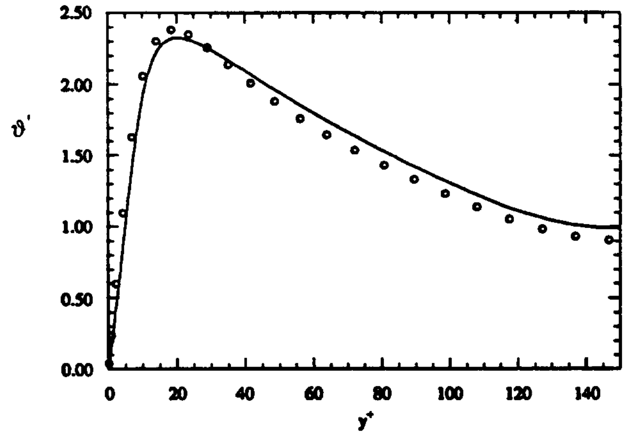

Figure 2 Profile of the rms temperature fluctuation in two-dimensional channel flow; wall coordinates; ∘ ∘ ∘ ∘ DNS (Kasagi 1992) ——— q_ϑ–ζ_ϑ model calculations

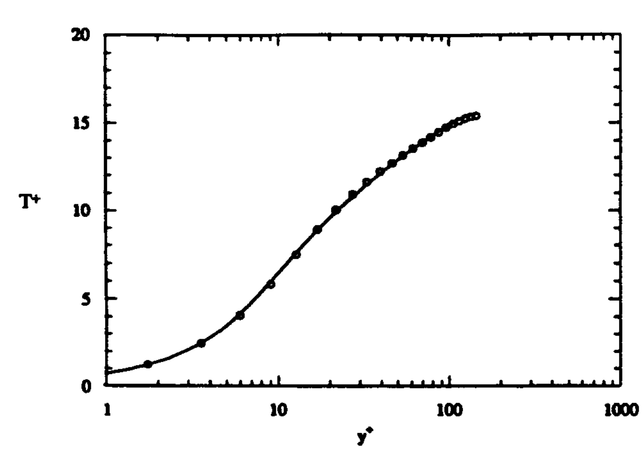

Figure 1 Mean temperature profile in two-dimensional channel flow; wall coordinates, ∘ ∘ ∘ ∘ DNS (Kasagi 1992) ——— q_ϑ–ζ_ϑ model calculations

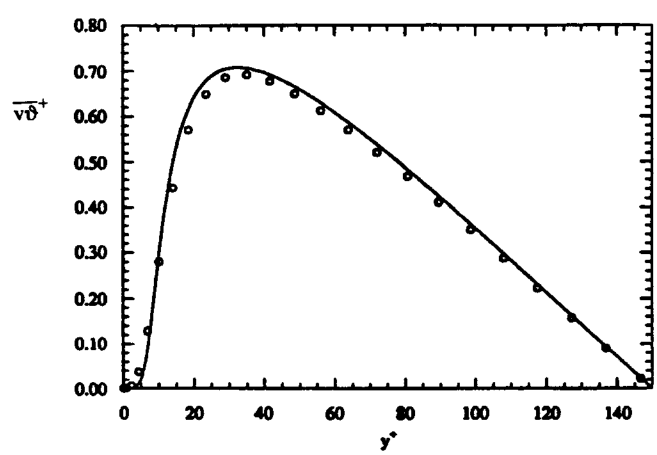

Figure 3 Profile of the temperature flux in two-dimensional channel flow; wall coordinates ∘ ∘ ∘ ∘ DNS (Kasagi 1992) ——— q_ϑ–ζ_ϑ model calculations

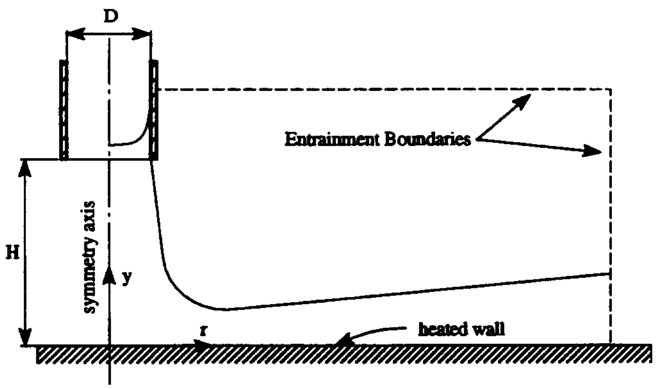

Figure 4 The impinging jet and solution domain

savings in CPU time reported by Gibson and Dafa'Alla (1995) for channel and boundary-layer flows.

Results

The region of interest is the flow in the vicinity of the stagnation point; the published profile measurements did not extend beyond $r = 3D$. Figure 5 shows that mean velocity distributions near the stagnation point are quite accurately predicted by both the q–ζ and Launder–Sharma k–ε models, but that the accuracy of the latter deteriorates with increasing radial distance. Measured and predicted values of the radial wall jet thickness are compared in Figure 6. Here $y_{1/2}$ is defined as the distance from the wall to the point in the outer flow where the velocity is one half of the maximum value. In this case, the growth rate is significantly underestimated in the q–ζ model calculations; the k–ε model gives approximately the right spreading rate in the far field but returns excessive thickness near the origin.

Figure 7 shows differences in the measured and calculated shear-stress profiles which would be surprising had we not seen similar results obtained by Craft et al. (1993) using the k–ε model. Craft et al. attributed the discrepancy to excessive turbulence production calculated in the vicinity of the stagnation point. We concur with this diagnosis and have verified that this fault is to some extent cured by the Yap (1978) correction. This useful "fix" limits the growth of the turbulence length scale by introducing the following wall-dependent source term in the ε-equation (or its equivalent in the ζ-equation):

$$S_\varepsilon = 0.83\left(\frac{l}{c_l y} - 1\right)\left(\frac{l}{c_l y}\right)^2 \frac{\tilde{\varepsilon}^2}{k} \tag{19}$$

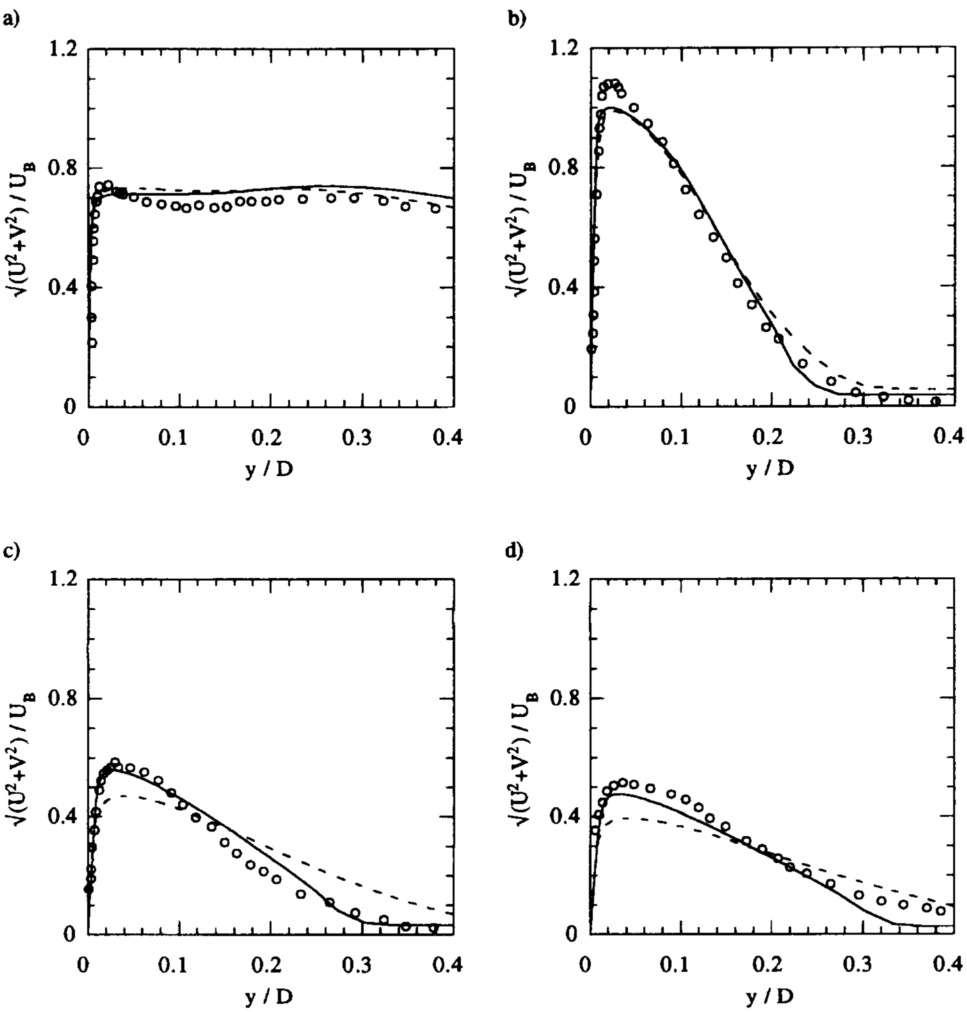

Figure 5 Impinging jet: profiles of the mean velocity: ∘ ∘ ∘ measurements (Cooper et al. 1993); (a) r/D = 0.5; (b) r/D = 1.0; (c) r/D = 2.5; (d) r/D = 3.0; ——— calculated (q–ζ model); ---- calculated (k–ε model)

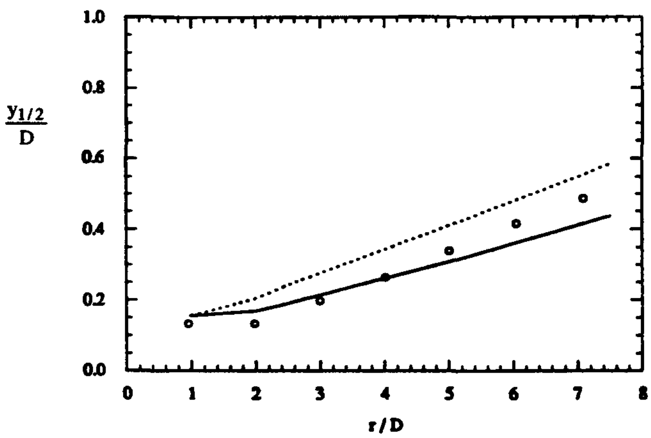

Figure 6 Impinging jet: development of the radial well jet: ∘ ∘ ∘ measurements (Cooper et al. 1993) ———calculated (q–ζ model); ---- calculated (k–ε model)

In local-equilibrium wall turbulence $l = c_l y$ and $S_\varepsilon = 0$. Craft et al. (1993) have shown that the Yap correction effectively halves the calculated heat transfer rate at the stagnation point, from $\mathrm{Nu} \cdot \mathrm{Re}^{-0.7}\mathrm{Pr}^{-0.4} \approx 0.32$ to 0.16 approximately, when it is used with a Reynolds-stress closure. The Nusselt number distributions presented in Figure 8 show that much the same result is obtained from the q_ϑ–ζ_ϑ heat-transfer model. The continuous line shows the results obtained when the q–ζ low-Reynolds number model is associated with a constant turbulent Prandtl number of 0.91. In the q_ϑ–ζ_ϑ model calculations shown by the dashed line Pr_t is recovered as an output from the calculations. The third curve on Figure 8 shows the combined effects of the Yap correction and the q_ϑ–ζ_ϑ model. The effect of the q_ϑ–ζ_ϑ model is to deliver low turbulent diffusivities and high turbulent Prandtl numbers in the neighbourhood of the stagnation point, as is shown in the Pr_t distributions displayed in Figure 9. To some extent, this result is accounted for by the choice of damping functions because, while $f_\lambda \to 0$ as $y \to 0$, f_μ is non-zero at the wall. Away from the wall however, high values of Pr_t are obtained, because small temperature time scales are recovered from the q_ϑ- and ζ_ϑ-equations. The limiting behaviour for t^*

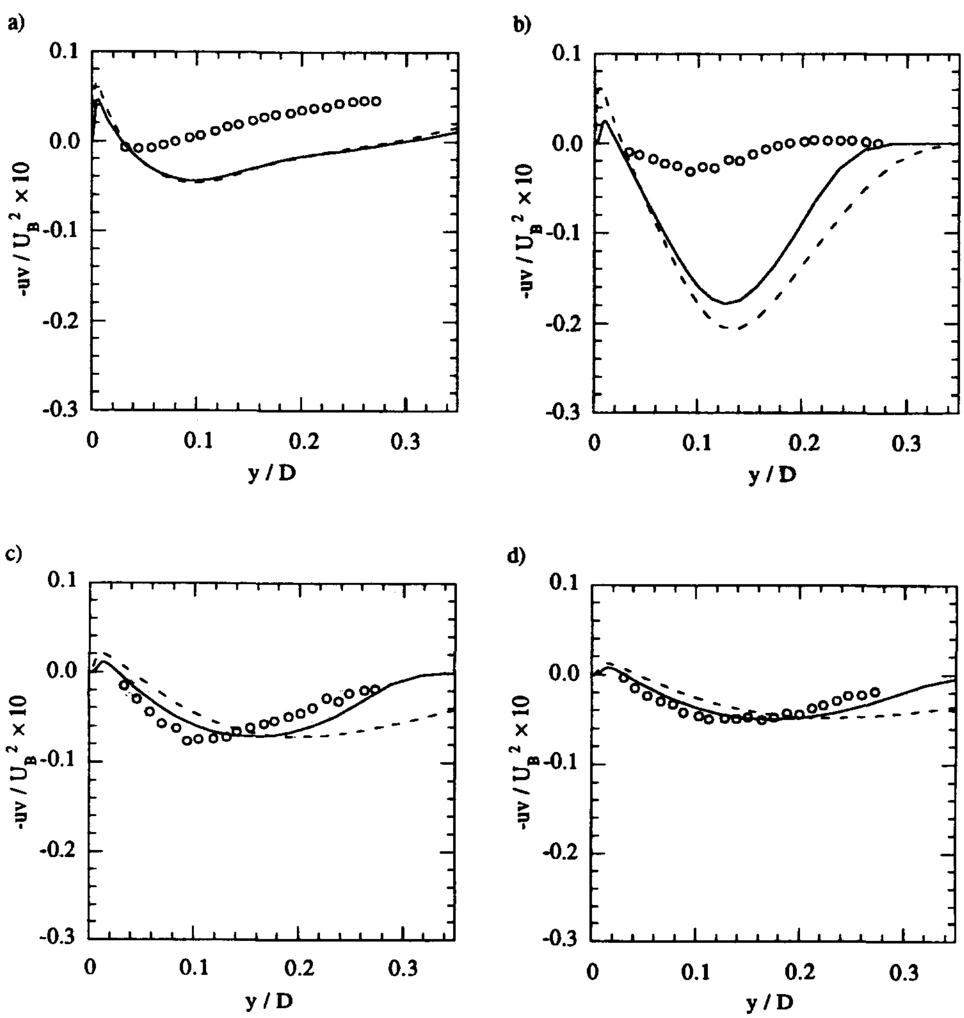

Figure 7 Impinging jet: profiles of the turbulent shear stress: ∘ ∘ ∘ measurements (Cooper et al. 1993); (a) r/D=0.5; (b) r/D=1.0; (c) r/D=2.5; (d) r/D=3.0; ———calculated (q–ζ model); ---- calculated (k–ε model)

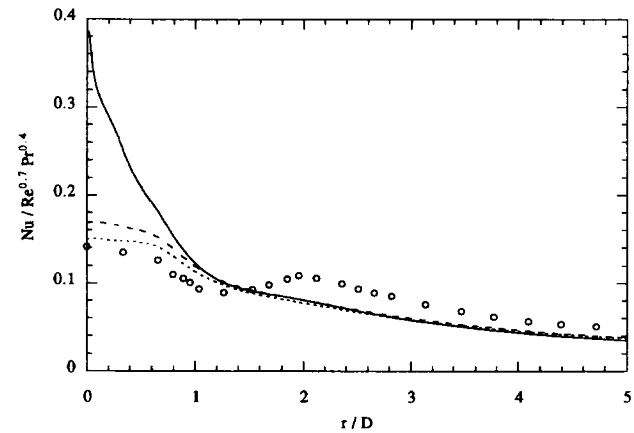

Figure 8 Impinging jet: heat transfer along the plate: ∘ ∘ ∘ measurements (Baughn and Shimizu 1989); calculations: —— (q–ζ model, Pr_t=0.91); ---- (q–ζ_θ and q_ϑ–ζ_ϑ models), · · · · (q–ζ model with Yap correction and q_ϑ–ζ_θ model)

(Equation 15) is $t^* \to 2t$ as $t_\vartheta \to \infty$; $t^* \to 4t_\vartheta \to 0$ as $t_\vartheta \to 0$. The results may be compared with those of Craft et al. who, using a Reynolds-stress closure for the velocity field, uniform Pr_t, and the Yap correction, managed to reproduce the secondary maximum shown by the data points at $x = 2D$.

Conclusions

Calculations on the q–ζ low-Reynolds-number eddy-viscosity model give marginally better results than the corresponding k–ε model but also retain the deficiencies previously noted by Craft et al. (1993) with respect to impinging jet, the most important of which is the excessive turbulence-energy level calculated in the approach flow to the wall. High levels of k of q are associated with high shear-stress levels which then produce excessive wall-jet growth rates, at any rate in the near field where the measurements were made. Use of the equivalent q_ϑ–ζ_ϑ model for heat transfer produced dramatic improvements in the stagnation-point Nusselt number, partly because of slight inconsistency in the specification of the low-Reynolds number diffusivity damping function in the q_ϑ–ζ_ϑ model (a defect to be remedied in further calculations) and partly because it seems that broadly correct values of q_ϑ and ζ_ϑ are delivered by the modelled transport equations, although we have no experimental data to support this assertion.

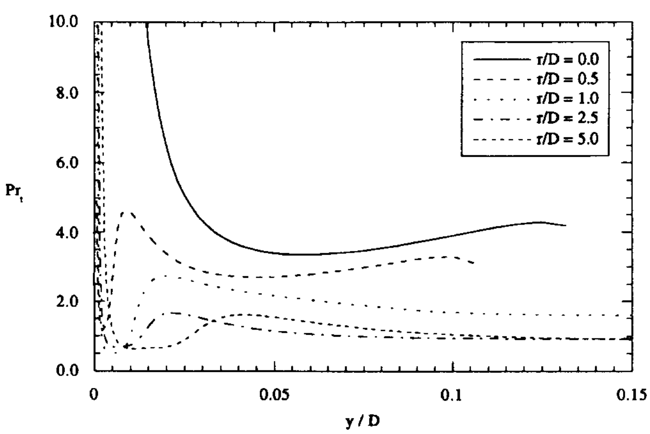

Figure 9 Impinging jet: turbulent Prandtl number in the vicinity of the stagnation point calculated with the q–ζ and q_ϑ–ζ_ϑ models

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